Microalgae bio-reactive façade: open data for nation-scale production potential assessment Supplementary materials

Victor Pozzobon[1] ✉

[1]LGPM, CentraleSupélec, Université Paris-Saclay, Centre Européen de Biotechnologie et de Bioéconomie (CEBB), 3 rue des Rouges Terres 51110 Pomacle, France

Thermal model

The model is based on a heat balance between the biofaçade reservoir and its surrounding. It comprises:

- incident direct solar radiation (Φ_{Sun}, split into infrared radiation and visible light, Eq. 1). $\Phi_{Sun,Vis}$, being determined by the model proposed by the Illuminating Engineering Society (1), accounts for 48.7 % of total sun power (2), which allows to derive $\Phi_{Sun,IR}$ (Eq. 2),
- sky radiation (Eq. 3, 4, 5, and 6), calculated using a sky temperature model (3–5),
- surrounding radiation (Eq. 9, 10, 11, 12 and 13), accounting for Urban Heat Island effect (6–9),
- controlled absorption of the visible part of the radiative heat flux (Eq. 7) and associated reflectivities (Eq. 8) (10),
- indoor radiation (Eq. 14 and 15) (11),
- indoor convection using resistance in series modeling approach (Eq. 16, 17, and 18) (4, 11, 12),
- outdoor convection using resistance in series modeling approach (Eq. 19, 20, 21, and 22) and Defraeye's correlation to assess wind contribution (Table 1) (13, 14),
- power originated from the gas sparged into the reservoir (Eq. 23).

Combined together, they govern the temporal evolution of the biofaçade reservoir temperature (Eq. 24).

$$\Phi_{Sun} = \Phi_{Sun,Vis} + \Phi_{Sun,IR} \tag{1}$$

$$\Phi_{Sun,IR} = (\frac{1}{0.487} - 1)\Phi_{Sun,Vis} = 1.05\Phi_{Sun,Vis} \tag{2}$$

$$\Phi_{Sky,Tot} = \sigma\epsilon_{Sky}T^4_{Sky} \tag{3}$$

$$\begin{aligned}\epsilon_{Sky}T^4_{Sky} &= 9.36575\ 10^{-6}(1-CC)T^6_{Air,Out} \\ &+ T^4_{Air,Out}CC[(1-0.84CC)(0.527+0.161\exp(8.45[1-\frac{273}{T_{Air,Out}}]))+0.84CC]\end{aligned} \tag{4}$$

$$\Phi_{Sky,Abs} = F_{Sky}(0.4\,\tau_{Sun} + 0.6\,\tau_{Sky,Abs})(\frac{3-2\alpha^2-\alpha}{3}(1-\eta_{ps})\Phi_{Sky,Vis} + (\Phi_{Sky,Tot} - \Phi_{Sky,Vis})) \tag{5}$$

$$\Phi_{Sky,Emi} = F_{Sky}\tau_{Sky,Emi}\sigma\epsilon_{mc}T^4_{mc} \tag{6}$$

$$\Phi_{Sun,Abs} = \tau_{Sun}(\frac{3-2\alpha^2-\alpha}{3}(1-\eta_{ps})\Phi_{Sun,Vis} + 1.05\Phi_{Sun,Vis}) \quad (7)$$

$$R_{interface} = \frac{1}{2}\left[\frac{\tan^2(\theta_i-\theta_r)}{\tan^2(\theta_i+\theta_r)} + \frac{\sin^2(\theta_i-\theta_r)}{\sin^2(\theta_i+\theta_r)}\right] \quad (8)$$

$$UHII = \max(T_{Urb} - T_{Rur}) \quad (9)$$

$$UHII = -0.54\,\overline{U} - 1.48\,\overline{CC} - 0.039\,\overline{Y} + 7.63 \quad (10)$$

$$T_{Rur} = (1-CC)(2.82 + 1.15T_{Air,Out}) + CC(1.33 + 1.00T_{Air,Out}) \quad (11)$$

$$\Phi_{Sur,Abs} = F_{Sur}\tau_{Sur,Abs}\sigma\epsilon_{Sur}T_{Sur}^4 \quad (12)$$

$$\Phi_{Sur,Emi} = F_{Sur}\tau_{Sur,Emi}\sigma\epsilon_{mc}T_{mc}^4 \quad (13)$$

$$\Phi_{In,Rad,Abs} = \tau_{In,Rad,Abs}\sigma\epsilon_{In}T_{In}^4 \quad (14)$$

$$\Phi_{In,Rad,Emi} = \tau_{In,Rad,Emi}\sigma\epsilon_{mc}T_{mc}^4 \quad (15)$$

$$h_{In,Conv,Free} = 2.04\,(\frac{H_{mc}}{H_{Ref,In}}(T_{pmma,In} - T_{Air,In}))^{0.23} \quad (16)$$

$$h_{In,Conv,Forced} = \frac{k_{Air}}{L_{Ref}}\;0.664\;Re_{Ref}^{1/2}Pr^{1/3} = 0.72\text{ W/m}^2\text{/K} \quad (17)$$

$$\Phi_{In,Conv,Net} = \frac{T_{Air,In} - T_{mc}}{\dfrac{1}{h_{In,Conv}} + \dfrac{e_{pmma}}{k_{pmma}}} \quad (18)$$

$$\overline{h_{Out,Conv,Free}} = \frac{k_{Air}}{E_{mc}}\left[0.825 + \frac{0.387\;Ra_L^{1/6}}{[1+(0.492\;Pr)^{9/16}]^{8/27}}\right]^2 \quad (19)$$

$$U_{Out} = U_{Station}\left(\frac{r_{Building}}{r_{Station}}\right)^{0.0706}\frac{\ln(\dfrac{E_{mc}+r_{Building}}{r_{Building}})}{\ln(\dfrac{E_{Station}+r_{Station}}{r_{Station}})} \quad (20)$$

$$h_{Out,Conv,Forced} = A_{\theta_{Wind}}U_{Out}^{B_{\theta_{Wind}}} \quad (21)$$

Wind incidence angle (θ_{Wind}, in degree)	$A_{\theta_{Wind}}$ (W/m^2/K)	$B_{\theta_{Wind}}$ (-)
0	4.90	0.86
30, 330	4.63	0.87
30, 300	4.25	0.88
90, 270	2.78	0.87
120, 240	1.44	0.83
150, 210	1.85	0.84
180	2.25	0.84

Table 1. Defraeye's correlation for different wind incidence angles on the facade. Couples of incidence angles tied to the same parameters originate from symmetry consideration

$$\Phi_{Out,Conv,Net} = \frac{T_{Air,Out} - T_{mc}}{\frac{1}{h_{Out,Conv}} + \frac{n_{Glaz} e_{pmma}}{k_{pmma}} + \frac{(n_{Glaz}-1) e_{Air}}{k_{Air}}} \quad (22)$$

$$P_{Gas,Net} = f H_{mc} w_{mc} e_{mc} \rho_{Gas} Cp_{Gas} (T_{Gas} - T_{mc}) \quad (23)$$

$$\begin{aligned} H_{mc} w_{mc} e_{mc} \rho_{Water} Cp_{Water} \frac{dT_{mc}}{dt} = & H_{mc} w_{mc} [\Phi_{Sun,Abs} + \Phi_{Sky,Abs} - \Phi_{Sky,Emi} + \Phi_{Sur,Abs} \\ & - \Phi_{Sur,Emi} + \Phi_{In,Rad,Abs} - \Phi_{In,Rad,Emi} + \Phi_{In,Conv,Net} \\ & + \Phi_{Out,Conv,Net}] + P_{Gas,Net} \end{aligned} \quad (24)$$

UHII(t) and UCII(t) reconstruction

Urban Heat Island temporal dynamic was reconstructed using the works of Montavez, Lai, and their coworkers (8, 15). Based on their observations, the UHII(t) dynamic was broken into three phases: 6 to 10 hours after sun dawn (taken as 10 hours based on Montavez observation in Europe), the UHII reaches its minimum (classically UHII_{Basal} = + 0.5 K). Then, the UHII rises to its maximum in about 5 hours and stabilizes overnight. Mathematically, they are described as a downward cosine (Eq. 25), an upward cosine (up to the value given by Eq. 10, Eq. 26), and an overnight plateau (at the value given by Eq. 10, Eq. 27). For each day, the new value of UHII was evaluated. Then, the days were chained together. Figure 1 compares experimental observations (15) and numerical reconstruction (UHII of the previous day: 2.75 K, basal level: +0.5 K, UHII for the night to come: +2.5 K). As one can see, the agreement can be deemed satisfactory.

$$UHII(t) = UHII_{Basal} + (UHII_{Previous\ day} - UHII_{Basal}) \frac{\cos(\pi \frac{t_{Solar} - t_{Sun\ dawn}}{10\ \text{hours}}) + 1}{2} \quad (25)$$

$$UHII(t) = UHII_{Basal} + (UHII_{Current\ day} - UHII_{Basal}) \frac{-\cos(\pi \frac{t_{Solar} - t_{Sun\ dawn} - 10\ \text{hours}}{5\ \text{hours}}) + 1}{2} \quad (26)$$

$$UHII(t) = UHII_{Current\ day} \quad (27)$$

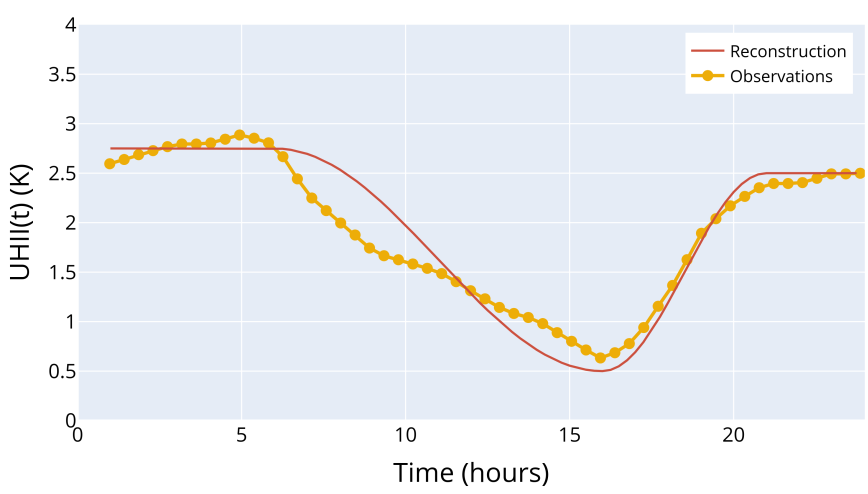

Fig. 1. Urban Heat Island temporal dynamics, observed (15) and reconstructed

In addition to the Urban Heat Island temporal dynamic, the Urban Cold Island temporal dynamic was also modeled. This phenomenon is the cold counterpart of the UHI. It happens transiently when the city mass is large enough to adsorb incident heat in such a large amount that the city center temperature is cooler than the closely rural area. This was identified in this work as when the UHII for a given day dropped below the basal level of +0.5 K. Mathematically, it is also described as three phases. First, a downward cosine (down to the negative value of the UHII, Eq. 28), an upward cosine (up to the basal value of +0.5 K, Eq. 29), and a plateau (at +0.5 K overnight (8), Eq. 30). With this model, Urban Heat and Cold Island phenomena can be described and chained indiscriminately.

$$UHII(t) = UHII_{Current\ day} + (UHII_{Previous\ day} - UHII_{Current\ day}) \frac{\cos(\pi \frac{t_{Solar} - t_{Sun\ dawn}}{10\ \text{hours}}) + 1}{2} \quad \textbf{(28)}$$

$$UHII(t) = UHII_{Current\ day} + (UHII_{Basal} - UHII_{Current\ day}) \frac{-\cos(\pi \frac{t_{Solar} - t_{Sun\ dawn} - 10\ \text{hours}}{5\ \text{hours}}) + 1}{2} \quad \textbf{(29)}$$

$$UHII(t) = UHII_{Basal} \quad \textbf{(30)}$$

Biological model

The model relies on an energy balance (Eq. 31) and the assumption that volume-averaged light intensity drives the photosynthetic efficiency and the acclimation dynamic of the culture. It features:

- actinic energy capture by the cells (Eq. 32),
- modulation by photosynthetic efficiency (Eq. 33), where growth is modeled as sigmoid (Eq. 34) based on data from (16) and (17),
- cells pigment modulation by the averaged light intensity, modeled as a first order response (Eq. 35) with assymptic values (Eq. 36), based on the data of (18) and (19) for the characteric times and (16) for the values (Table 2),
- modulation of the overall metabolism by the temperature (Eq. 37),
- and various refinement such as the differentiation of the microalgae optical properties between red, green, and blue part of the visible, the use of light and dark respiration, the indexation of the cell absorption coefficient based on their pigment content (the reader is kindly referred to the original article to know more about these augmentations (20)).

$$\underbrace{V\frac{dX}{dt}}_{\text{Cell mass variation}} = \underbrace{\frac{\Phi_{Abs,Vis}(I_0, X)}{HHV}\xi(I_{Average})}_{\text{Absorbed power and usage efficiency}} - \underbrace{VmeX}_{\text{Maintenance}} \quad \textbf{(31)}$$

$$I_{Average} = \frac{I_0}{\Sigma_{Abs} X L}(1 - exp(-\Sigma_{Abs} X L)) \quad \textbf{(32)}$$

$$\xi(I_{Average}) = \xi_0 \frac{4I_r}{I_{Average}}(\frac{1}{1 + exp(-I_{Average}/I_{Ref})} - \frac{1}{2}) \quad \textbf{(33)}$$

$$\mu_{Gross}(I_{Average}) = 2\mu_{Max}(\frac{1}{1 + exp(-I_{Average}/I_{Ref})} - \frac{1}{2}) \quad \textbf{(34)}$$

$$\frac{dZ_{Pig}(t)}{dt} = \frac{1}{\tau_{Z,Pig}}(Z_{Pig,Eq}(I_{Average}) - Z_{Pig}(t)) \quad \textbf{(35)}$$

$$Z_{Pig,Eq}(I_{Average}) = A\ exp(-\frac{I_{Average}}{aI_{Average} + b}) \quad \textbf{(36)}$$

Pigment	A (mg/g$_{DW}$)	a (-)	b (µmolPhoton/m²/s)	RMSE (mg/g$_{DW}$)
Chlorophyll *a*	25.2 ± 0.06	0.361 ± 0.001	56.7 ± 0.3	0.949
Chlorophyll *b*	12.0 ± 0.02	0.364 ± 0.000	59.4 ± 0.2	0.625
Lutein	6.53 ± 0.01	0.315 ± 0.000	17.6 ± 0.2	0.318

Table 2. Parameters describing the cell equilibrium pigment content, with 95 % confidence interval

$$\frac{\mu(T)}{\mu_{Max}} = \frac{(T - T_{Max})(T - T_{Min})^2}{(T_{Opt} - T_{Min})[(T_{Opt} - T_{Min})(T - T_{opt}) - (T_{Opt} - T_{Max})(T_{Opt} + T_{Min} - 2T)]} \quad \textbf{(37)}$$

Finally, the overall workflow is illustrated in Figure 2.

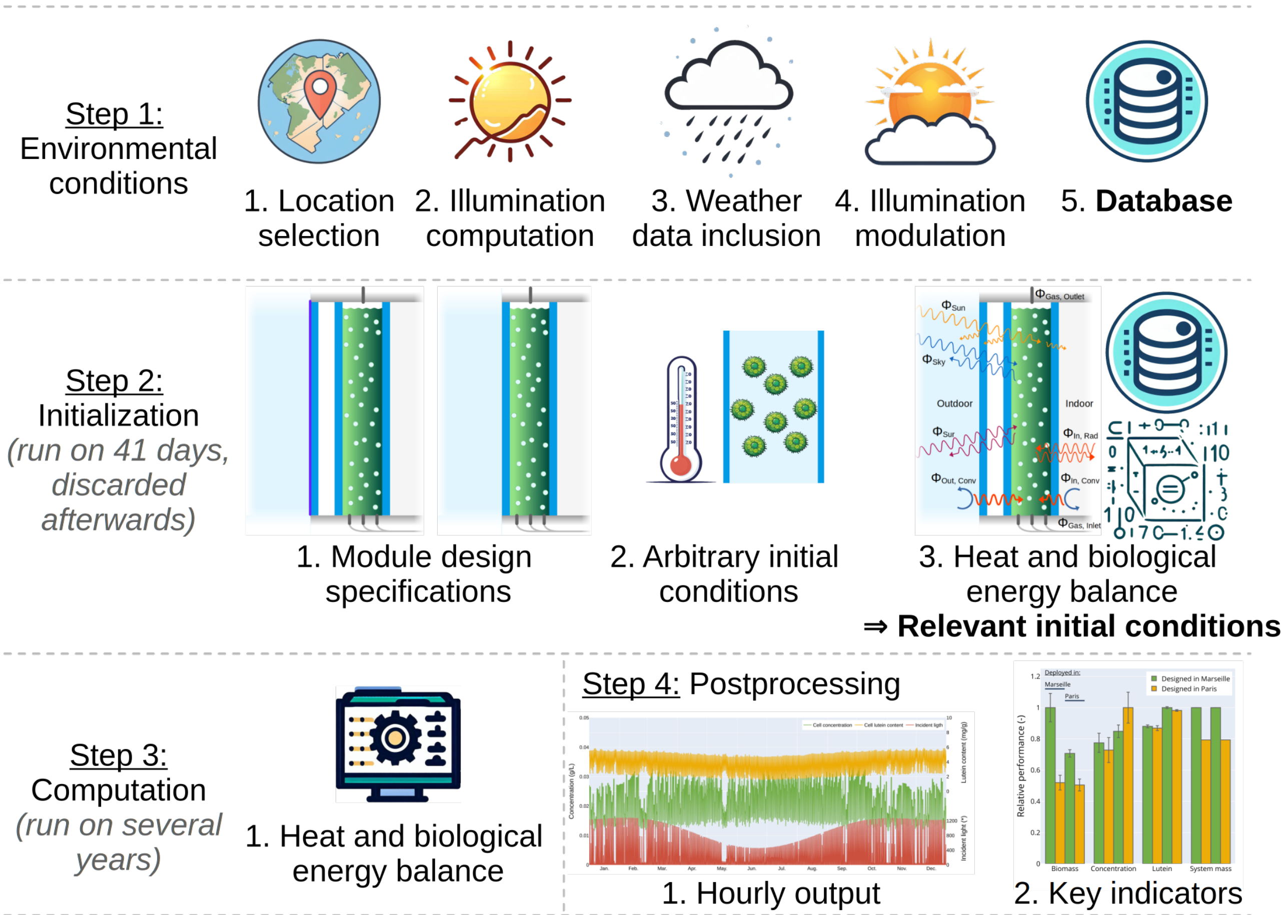

Fig. 2. Numerical workflow

Nomenclature

Latin symbols	Property	Unit
A	First parameter of Defraeye's correlation	-
B	Second parameter of Defraeye's correlation	$W/m^2/K$
CC	Cloud Cover factor	-
Cp	Specific heat	J/kg/K
E	Elevation above the ground	m
e	Thickness	m
F	View factor	-
f	Areation	VVM (Vessel Volume per Minute)
HHV	Higher Heating Value	MJ/kg
h	Convective heat transfer coefficient	$W/m^2/K$
k	Thermal conductivity	W/m/K
I_0	Incident photosynthetically active light intensity	µmolPhotonPAR/m^2/s
k	Conversion factor	µmolPhotonPAR/J
L	Characteristic length	m
me	Maintenance rate	1/day
n_X	Number of X	-
P	Power	W
Pr	Prandtl number	-
Re	Reynolds number	-
R	Reflectivity	-
Ra	Rayleigh number	-
r	Surface roughness	m
T	Temperature	°C in the text / K in formulas
t	Time	s
U	Velocity	m/s
UHII	Urban Heat Island Intensity	K
w	Width	m
X	Cell concentration	kg/m^3
Y	Relative humidity	%
Z	Pigment content	mg/g

Greek symbols	Property	Unit
α	Green light transmitted fraction	-
ϵ	Emissivity	-
η	Efficiency	-
θ	Angle	rad
μ	Microalgae growth rate	1/day
ξ	Photosynthetic efficiency	-
ρ	Density	kg/m^3
Σ	Cross section	m^2/kg
σ	Boltzmann's constant	$W/m^2/K^4$
τ_Z	Characteristic time	s
τ	Transmission	-
Φ	Heat flux	W/m^2

Subscript	Description
Abs	Absorbed by the culture
Air	Air, indoor or outdoor
Average	Average over the volume
Building	Building hosting the facade
Conv	Convective-conductive
Emi	Emitted
Forced	Forced convection
Free	Free convection
Gas	Sparged gas
Gross	Gross
Glaz	Glazing
i	Incidence
In	Indoor
Interface	Interface
IR	Infrared
Max	Maximum
mc	Microalgae culture
Min	Minimum
Net	Net exchange
Opt	Optimal
Out	Outdoor
Pig	Pigment
pmma	PolyMethyl MethAcrylate
ps	Photosynthesis
r	Refraction
Rad	Radiative
Ref	Reference
Rur	Rural
Sky	Sky
Station	Meteorological station
Sun	Sun
Sur	Surrounding
Tot	Total
Urb	Urban
Vis	Visible
Water	Water
Wind	Wind
0	At I = 0 μmolPhotonPAR/m^2/s

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